

Suppose that the height of a Ferris wheel seat changes in a pattern that can be modeled by the function $h(t) = 25 \sin t + 30$, where time is in minutes and height is in meters.

What are the period and amplitude of $h(t)$? What do those values tell about the motion of the Ferris wheel.

If a seat starts out in the "3 o'clock" position, how long will it take the seat to return to that position? At what times will it revisit that position?

Suppose the height (in meters) of seats on different Ferris wheels changes over time (in minutes) according to the functions give below. For each function:

- Find the height of the seat when the motion of the wheel begins $t=0$
- Find the amplitude of $h(t)$. Explain what it tells you about motion of the wheel.

• Per • max min

$$y = A \cos Bt + C$$

$$y = A \sin Bt + C$$

A = Amplitude

$$\text{Per} = \frac{2\pi}{B}$$

$$\frac{2\pi}{1} \quad \frac{2\pi}{1} \cdot \frac{1}{2}$$

$$\frac{2\pi}{1} \cdot \frac{2}{1} = 4\pi$$

$$\frac{2\pi}{3} \quad \frac{2\pi}{1} \cdot \frac{3}{2}$$

$$\frac{2\pi}{1} \cdot \frac{2}{3} = \frac{4\pi}{3}$$

$$h(t) = 15 \sin 0.5t + 17$$

$$h(0) = 15 \sin 0.5(0) + 17 = 17 \text{ m}$$

$$\text{Amp} = 15 - \text{Radius}$$

$$\text{Per} = \frac{2\pi}{B} = \frac{2\pi}{.5} = 4\pi$$

$$\text{max} = 32 \quad = 12.56$$

$$\text{min} = 2 \quad 12 \text{ min } 33 \text{ sec}$$

$$h(t) = 12 \sin 1.5t + 13$$

$$h(0) = 12 \sin 1.5(0) + 13 = 13$$

$$\text{Per} = \frac{2\pi}{1.5} = \frac{4\pi}{3}$$

$$= 4.19 \quad 4 \text{ min } 11 \text{ sec}$$

$$h(t) = 24 \cos 2t + 27$$

$$h(0) = 24 \cos 2(0) + 27 = 51 \text{ m}$$

$$\text{Amp} = 24 \rightarrow \text{Radius}$$

$$\text{Per} \frac{2\pi}{B} = \frac{2\pi}{2} = \pi \rightarrow$$

$$\text{max} = 51 \text{ m}$$

$$\text{min} = 3 \text{ m}$$

$$h(t) = -12 \cos t + 14$$

$$h(0) = -12 \cos(0) + 14 = 2 \text{ m}$$

$$\text{Amp} = 12 \rightarrow \text{Radius}$$

$$\text{Per} \frac{2\pi}{B} = \frac{2\pi}{1} = 6.28 \rightarrow 6 \text{ min } 17 \text{ sec}$$

$$(1.19)(60)$$

$$24 \cos(0)$$

$$24(1)$$

$$3.14$$

$$3 \text{ min } 9 \text{ sec}$$

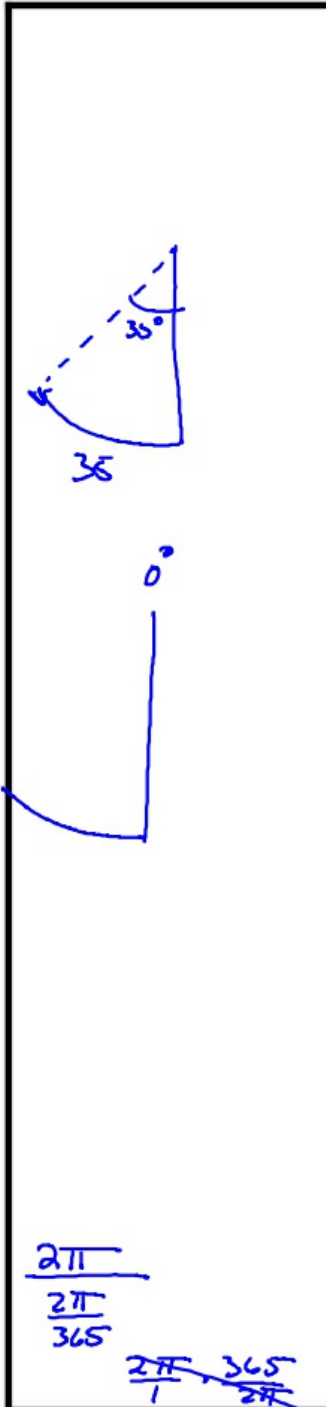
$$(1.19)(60)$$

$$\text{max} = 25 \text{ m}$$

$$\text{min} = 1 \text{ m}$$

$$\text{max} \rightarrow 26 \text{ m}$$

$$\text{min} \rightarrow 2 \text{ m}$$



Pendulums are among the simplest but most useful examples of periodic motion. Once set in motion, the arm of the pendulum swings left and right on a vertical axis. The angle of displacement from vertical is a periodic function of time that depends on the length of the pendulum and its initial release point.

Suppose that the function $d(t) = 35 \cos 2t$ give the displacement from vertical (in degrees) of the tire swing pendulum shown at the right as a function of time (in seconds).



What are the amplitude and period of $d(t)$? What does each tell you about the motion of the swing?

$Amp = 35$ $3.14 \rightarrow sec$

$Per = \frac{2\pi}{B} = \frac{2\pi}{2} = \pi \rightarrow 3.14 \text{ sec to make one complete swing}$

If the motion of a different swing is modeled by $f(t) = 45 \cos \pi t$, what are the amplitude and period of $f(t)$? What does each tell you about the motion of the swing?

$Amp = 45$ $Per = \frac{2\pi}{B} = \frac{2\pi}{\pi} = 2 \text{ sec.}$

Why does it make sense to use variation of the circular function $\cos t$ to model pendulum motion?

What function $g(t)$ would model the motion of a pendulum that is released from a displacement of 18° right of vertical and swings with a frequency of 0.25 cycles per second (a period of 4 seconds)?

$A = 18^\circ$ $Per = \frac{2\pi}{B}$
 $Per = 4$ $B(4) = \left(\frac{2\pi}{B}\right)B$
 $g(t) = 18 \cos \frac{\pi}{2} t$ $4B = 2\pi$
 $B = \frac{2\pi}{4}$
 $= \frac{\pi}{2}$

At every location on Earth, the number of hours of daylight varies with the season in a predictable way. One convenient way to model that pattern of change is to measure time in days, beginning with spring equinox (about March 21st) as $t = 0$. With that frame of reference, the number of daylight hours in Boston, Massachusetts is given by $d(t) = 3.5 \sin \frac{2\pi}{365} t + 12.5$.

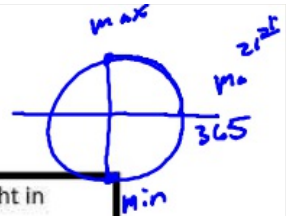
What are the amplitude and period of the $d(t)$? What do those values tell about the pattern of change in daylight during a year in Boston?

$P = 365 \text{ days in year}$

$\frac{2\pi}{365}$
 $\frac{2\pi}{365}$
 ~~$\frac{2\pi}{1}, \frac{365}{2\pi}$~~

$Amp = 3.5 \rightarrow \frac{max - min}{2}$

$$3.5 \sin \frac{2\pi}{365} t + 12.5$$



What are the maximum and minimum numbers of hours of daylight in Boston? What times in the year do they occur?

max → 16 hours June 21st
 min → 9 hours Dec 21st

If the function giving the number of daylight hours in Point Barrow Alaska, had the form $f(t) = a \sin bt + c$, how would you expect the values of a , b , and c to be related to the corresponding numbers in the rule giving daylight hours in Boston?

$b = \text{No change}$

9/day
 273
 210
 9

Why does it make sense that the function giving daylight hours at points on Earth should involve the circular function $\sin t$?